647510 **MSC2MatC204x** **Seat No :_________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
May/June-2022 (NEW COURSE)
METHODS IN PARTIAL DIFFERENTIAL EQUATIONS – 2 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1 (A) Answer any Two out of Three **[10]**

(1) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$.

(2) Define Pfaffian Differential equation. When it is exact? Justify your answer.

(3) Check whether $(1 + yz)dx + (xz - x^2)dy - (1 + xy)dz = 0$ is integrable or not. If it is integrable then find its solution.

Q-1 (B) Answer any One out of Two **[4]**

(1) Solve: $y^2p - xyq = x(z - 2y)$ using an appropriate method.

(2) Let $X = (P, Q, R)$, where P, Q and R are functions of x, y and z. Then prove that, if Pfaffian differential equation $Pdx + Qdy + Rdz = 0$ is integrable then, $X \cdot curl\ (X) = 0$.

Q-2 (A) Answer any Two out of Three **[10]**

(1) Obtain a partial differential equation for the set of all spheres of radius a whose center lie along z-axis.

(2) Find the integral surface of $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} + z^2 = 0$ which passes through the hyperbola $xy = x + y, z = 1$.

(3) Find the surface which intersects the surface of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

Q-2 (B) Answer any One out of Two **[4]**

(1) Eliminate the arbitrary function F from the equation $z = xy + F(x^2 + y^2)$ and find the corresponding partial differential equation.

(2) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$$

Q-3 (A) Answer any Two out of Three **[10]**

(1) Find a complete integral of $q = (z + px)^2$ using Charpit's method.

(2) Find a complete integral of $2p_1x_1x_3 + 3p_2x_3^2 + p_2^2p_3 = 0$, where $p_1 = \frac{\partial z}{\partial x_1}, p_2 = \frac{\partial z}{\partial x_2}$ and $p_3 = \frac{\partial z}{\partial x_3}$ using Jacobi's method.

(3) Show that a complete integral of the equation $f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right) = 0$ is $u = ax + by + \theta(a, b)z + c$, where a, b and c are arbitrary constants and $f(a, b, \theta) = 0$.

Q-3 (B) Answer any One out of Two **[4]**

(1) Solve $p^2 + q^2 = k^2$, where k is a constant.

(2) Find a complete integral of $p(1 + q^2) = q(z - 1)$.

Q-4 (A) Answer any Two out of Three **[10]**

(1) Find the general solution of $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$.

(2) For $(D^3 - 2D^2D' - DD' + 2D'^3)z = e^{x+y}$, find the general solution.

(3) Solve: $(x^2D^2 - y^2D'^2)z = xy$.

Q-4 (B) **Answer any One out of Two** **[4]**

(1) If u is the complementary function and z' a particular integral of a linear partial differential equation $F(D, D') = f(x, y)$, then prove that $u + z'$ is a general solution of the equation.

(2) Find particular integral of $(2D^2 - 3D + D')z = \sin(2x - y)$.

Q-5 (A) **Answer any Two out of Three** **[10]**

(1) Let $X = (P, Q, R)$, where P, Q and R are functions of x, y and z. Let $0 \neq \mu = \mu(x, y, z)$ be any function. Then, prove that $X \cdot curl\,(X) = 0$ if and only if $\mu X \cdot curl\,(\mu X) = 0$.

(2) Find the integral equation of linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.

(3) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

Q-5 (B) **Answer any One out of Two** **[4]**

(1) Find the complete integral of the equation

$(px + qy - z)^2 = 1 + p^2 + q^2$.

(2) Obtain partial differential equation corresponding to $z = f(x + ay) + g(x - ay)$, where a is a constant.

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